Laplace Transforms - Written Answers

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PART 11
FIRST PRINCIPLES

1) $\mathcal{L}\{1\} = \int_0^\infty e^{-st} \cdot 1 \, dt$

$$= \int_0^\infty e^{-st} \, dt$$

$$= \lim_{A \to \infty} \int_0^A e^{-st} \, dt$$

$$= \lim_{A \to \infty} \left[-\frac{1}{s} e^{-st} \right]_0^A$$

$$= \lim_{A \to \infty} \left(-\frac{1}{s} e^{-sA} + \frac{1}{s} e^{-s0} \right)$$

$\frac{1}{s} e^0$

$\frac{1}{s} \cdot 1$

$$= \lim_{A \to \infty} \left(-\frac{1}{se^{sA}} + \frac{1}{s} \right)$$

int

$e^x \to e^x$

$e^{kx} \to \frac{1}{k} e^x$

$\frac{1}{se^{s\infty}}$

$\frac{1}{1{,}000} = 0.001$

$\frac{1}{1{,}000{,}000} = 0.000001$

$\frac{1}{\infty} \to 0$

$$= 0 + \frac{1}{s}$$

$$= \frac{1}{s}$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$

2) $\mathcal{L}\{6\} = \int_0^\infty e^{-st} \cdot 6 \, dt$

$= 6\int_0^\infty e^{-st} \, dt$

$= \lim_{A \to 0} 6\int_0^A e^{-st} \, dt$

$= \lim_{A \to 0} 6 \left[-\frac{1}{s} e^{-st} \right]_0^A$

$= \lim_{A \to 0} 6 \left(-\frac{1}{s} e^{-sA} - -\frac{1}{s} e^{-s0} \right)$

$e^0 = 1$

$= \lim_{A \to 0} 6 \left(-\frac{1}{se^{sA}} + \frac{1}{s} \right)$

$= \lim_{A \to 0} 6 \left(\frac{1}{s} \right)$

$= \lim_{A \to 0} \frac{6}{s}$

$\mathcal{L}\{6\} = \frac{6}{s}$ ✓

$$3)\ \mathcal{L}\{6t\} = \int_0^\infty e^{-st}\, 6t \, dt \qquad \int_0^\infty e^{-st} f(t)\, dt$$

$$= 6\int_0^\infty e^{-st} t \, dt$$

$$= \lim_{A\to\infty} 6\int_0^A t e^{-st}\, dt$$

BY PARTS

$$u = t \qquad v = -\frac{1}{s}e^{-st}$$

$$\frac{du}{dt} = 1 \qquad \frac{dv}{dt} = e^{-st}$$

L
A
T
E

$$\int u\frac{dv}{dt}\,dt = uv - \int v\frac{du}{dt}\,dt$$

$$= t\cdot -\frac{1}{s}e^{-st} - \int -\frac{1}{s}e^{-st}\cdot 1 \, dt$$

$$= \frac{-t}{se^{st}} - -\frac{1}{s}\int e^{-st}\, dt$$

$$= \frac{-t}{se^{st}} + \frac{1}{s}\left(-\frac{1}{s}e^{-st}\right)$$

$$= \frac{-t}{se^{st}} - \frac{1}{s^2e^{st}}$$

$$= 6\left[\frac{-t}{se^{st}} - \frac{1}{s^2e^{st}}\right]_0^A$$

Could do this, but better to factor numericals in @ end.

$$= \left[\frac{-6t}{se^{st}} - \frac{6}{s^2e^{st}}\right]_0^A$$

$$= 6\left[\left(\frac{-A}{se^{sA}} - \frac{1}{s^2 e^{sA}}\right) - \left(\frac{-0}{se^{s(0)}} - \frac{1}{s^2 e^{s(0)}}\right)\right]$$

practically $0 - 0 = 0$

$= \frac{1}{e^{\infty}} = \frac{\text{very small number}}{\text{infinitely bigger number}} \Rightarrow 0$

$= \frac{-\infty}{e^{\infty}} \Rightarrow \frac{\text{infinite number}}{\text{infinitely bigger number}} \Rightarrow 0$

$\boxed{n^0 = 1}$

$$= 6\left[\ 0 \quad - \left(\frac{0}{s(1)} - \frac{1}{s^2(1)}\right)\right]$$

$$= 6\left[0 + \frac{1}{s^2}\right]$$

$$= 6\left[\frac{1}{s^2}\right]$$

$$= 0 + \frac{6}{s^2}$$

$$= \frac{6}{s^2}$$

✓

4) $\mathcal{L}\{t^5\} = \int_0^\infty e^{-st} \cdot t^5 \, dt$

$$\int_0^\infty e^{-st} f(t) \, dt$$

$$= \lim_{A \to \infty} \underbrace{\int_0^A e^{-st} \cdot t^5 \, dt}$$

$u = t^5 \qquad v = -\frac{1}{s} \cdot e^{-st}$

$\frac{du}{dt} = 5t^4 \qquad \frac{dv}{dt} = e^{-st}$

L
A
T
E

$$\int u \frac{dv}{dt} dt = uv - \int v \frac{du}{dt} dt$$

$$= t^5 \cdot -\frac{1}{s} e^{-st} - \int -\frac{1}{s} \cdot e^{-st} \cdot 5t^4 \, dt$$

$$= -\frac{t^5}{se^{st}} + \frac{5}{s} \underbrace{\int e^{-st} t^4 \, dt}$$

$u = t^4 \qquad v = -\frac{1}{s} \cdot e^{-st}$

$\frac{du}{dt} = 4t^3 \qquad \frac{dv}{dt} = e^{-st}$

$$= t^4 \cdot -\frac{1}{s} e^{-st} - \int -\frac{1}{s} e^{-st} \cdot 4t^3 \, dt$$

$$= -\frac{t^4}{se^{st}} + \frac{4}{s} \underbrace{\int e^{-st} \cdot t^3 \, dt}$$

$u = t^3 \qquad v = -\frac{1}{s} e^{-st}$

$\frac{du}{dt} = 3t^2 \qquad \frac{dv}{dt} = e^{-st}$

$$\int u \frac{dv}{dt} dt = uv - \int v \frac{du}{dt} dt$$

$$t^3 \cdot -\frac{1}{s} e^{-st} - \int -\frac{1}{s} e^{-st} \cdot 3t^2 \, dt$$

$$-\frac{t^3}{se^{st}} + \frac{3}{s}\underbrace{\int e^{-st}\cdot t^2\,dt}$$

$u = t^2 \qquad v = -\frac{1}{s}e^{-st}$

$\frac{du}{dt} = 2t \qquad \frac{dv}{dt} = e^{-st}$

$$t^2\cdot -\frac{1}{s}e^{-st} - \int -\frac{1}{s}e^{-st}\cdot 2t\,dt$$

$$-\frac{t^2}{se^{st}} + \frac{2}{s}\underbrace{\int e^{-st}\cdot t\,dt}$$

$\int u\frac{dv}{dt}\,dt = uv - \int v\frac{du}{dt}\,dt$

$u = t \qquad v = -\frac{1}{s}e^{-st}$

$\frac{du}{dt} = 1 \qquad \frac{dv}{dt} = e^{-st}$

$$t\cdot -\frac{1}{s}e^{-st} - \int -\frac{1}{s}e^{-st}\cdot 1\,dt$$

$$-\frac{t}{se^{st}} + \frac{1}{s}\int e^{-st}\,dt$$

$$\frac{-t}{se^{st}} + \frac{1}{s}\cdot -\frac{1}{se^{st}}$$

$$\frac{-t}{se^{st}} - \frac{1}{s^2e^{st}}$$

$$= \left[-\frac{t^5}{se^{st}} + \frac{5}{s}\left(-\frac{t^4}{se^{st}} + \frac{4}{s}\left(-\frac{t^3}{se^{st}} + \frac{3}{s}\left(-\frac{t^2}{se^{st}} + \frac{2}{s}\left(-\frac{t}{se^{st}} - \frac{1}{s^2 e^{st}} \right)\right)\right)\right) \right]_0^A$$

$t=A$

$$= 0 + \frac{5}{s}\left(0 + \frac{4}{s}\left(0 + \frac{3}{s}\left(0 + \frac{2}{s}\left(0 - 0\right)\right)\right)\right)$$

$$= 0$$

$t=0$

$$= 0 + \frac{5}{s}\left(0 + \frac{4}{s}\left(0 + \frac{3}{s}\left(0 + \frac{2}{s}\left(0 - \frac{1}{s^2}\right)\right)\right)\right)$$

$$= \frac{5}{s}\left(\frac{4}{s}\left(\frac{3}{s}\left(\frac{2}{s}\left(-\frac{1}{s^2}\right)\right)\right)\right)$$

$$= -\frac{120}{s^6}$$

$$= 0 - -\frac{120}{s^6}$$

$$= \underline{\underline{\frac{120}{s^6}}} \checkmark$$

IDENTITY:

$$\mathcal{L}\{kt^n\} = \frac{k(n!)}{s^{n+1}}$$

PART III - USING THE TABLE

5) $\mathcal{L}\{5\} = 5\mathcal{L}\{1\}$

$= 5\left(\frac{1}{s}\right)$

$= \frac{5}{s}$

$f(t)$	$F(s)$
1	$\frac{1}{s}$

6) $\mathcal{L}\{\cos 4t\} = \frac{s}{s^2+4^2}$

$a=4$

$= \frac{s}{s^2+16}$

$\cos(at)$	$\frac{s}{s^2+a^2}$

7) $\mathcal{L}\{t^5\} = \frac{5!}{s^{5+1}}$

$= \frac{120}{s^6}$

t^n	$\frac{n!}{s^{n+1}}$

8) $\mathcal{L}\{\sin 5t\} = \frac{5}{s^2+5^2}$

$a=5$

$= \frac{5}{s^2+25}$

$\sin(at)$	$\frac{a}{s^2+a^2}$

$\cosh(at)$	$\frac{s}{s^2 - a^2}$

9) $\mathcal{L}\{\cosh(8t)\} = \frac{s}{s^2 - 8^2}$

$a = 8$

$= \frac{s}{s^2 - 81}$

10) $\mathcal{L}\{t^3 - 8t^2 + 1\}$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(at)$	$\frac{a}{s^2 + a^2}$
$\cos(at)$	$\frac{s}{s^2 + a^2}$
$\sinh(at)$	$\frac{a}{s^2 - a^2}$
$\cosh(at)$	$\frac{s}{s^2 - a^2}$

$= \mathcal{L}\{t^3\} - \mathcal{L}\{8t^2\} + \mathcal{L}\{1\}$

$= \mathcal{L}\{t^3\} - 8\,\mathcal{L}\{t^2\} + \mathcal{L}\{1\}$

$= \frac{3!}{s^{3+1}} - 8\left(\frac{2!}{s^{2+1}}\right) + \frac{1}{s}$

$= \frac{6}{s^4} - 8\left(\frac{2}{s^3}\right) + \frac{1}{s}$

$= \frac{6}{s^4} - \frac{16}{s^3} + \frac{1}{s}$

11) $\mathcal{L}\{2t^2 - \frac{1}{3}\}$

$= \mathcal{L}\{2t^2\} - \mathcal{L}\{\frac{1}{3}\}$

$= 2\mathcal{L}\{t^2\} - \frac{1}{3}\mathcal{L}\{1\}$

$= 2\left(\frac{2!}{s^{2+1}}\right) - \frac{1}{3}\left(\frac{1}{s}\right)$

$= \frac{4}{s^3} - \frac{1}{3s}$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$

12) $\mathcal{L}\{\frac{1}{7}t^3 - 4\sin(9t) - 6\}$

$= \mathcal{L}\{\frac{1}{7}t^3\} - \mathcal{L}\{4\sin(9t)\} - \mathcal{L}\{6\}$

$= \frac{1}{7}\mathcal{L}\{t^3\} - 4\mathcal{L}\{\sin(9t)\} - 6\mathcal{L}\{1\}$

$= \frac{1}{7}\left(\frac{3!}{s^{3+1}}\right) - 4\left(\frac{9}{s^2+9^2}\right) - 6\left(\frac{1}{s}\right)$

$= \frac{6}{7s^4} - \frac{36}{s^2+81} - \frac{6}{s}$

13) $\mathcal{L}\{5\cos(3t) - 6t^3\}$

$= 5\mathcal{L}\{\cos(3t)\} - 6\mathcal{L}\{t^3\}$

$= 5\left(\dfrac{s}{s^2+3^2}\right) - 6\left(\dfrac{3!}{s^{3+1}}\right)$

$= \dfrac{5s}{s^2+9} - \dfrac{36}{s^4}$

$f(t)$	$F(s)$
1	$\dfrac{1}{s}$
t^n	$\dfrac{n!}{s^{n+1}}$
$\sin(at)$	$\dfrac{a}{s^2+a^2}$
$\cos(at)$	$\dfrac{s}{s^2+a^2}$
$\sinh(at)$	$\dfrac{a}{s^2-a^2}$
$\cosh(at)$	$\dfrac{s}{s^2-a^2}$

PART IV - Step Function

$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$\sin(at)\cdot u(t)$	$\frac{a}{s^2+a^2}$
$\cos(at)\cdot u(t)$	$\frac{s}{s^2+a^2}$
$\sinh(at)\cdot u(t)$	$\frac{a}{s^2-a^2}$
$\cosh(at)\cdot u(t)$	$\frac{s}{s^2-a^2}$

14) $\mathcal{L}\{3u(t)\} = 3\left(\frac{1}{s}\right)$

$$= \frac{3}{s}$$

15) $\mathcal{L}\{\cos(4t)\cdot u(t)\} = \frac{s}{s^2+4^2}$

$$= \frac{s}{s^2+16}$$

16) $\mathcal{L}\{(6t^4 - 7\sin(11t))\cdot u(t)\}$

$$= 6\mathcal{L}\{t^4\} - 7\mathcal{L}\{\sin(11t)\}$$

$$= 6\left(\frac{4!}{s^{4+1}}\right) - 7\left(\frac{11}{s^2+11^2}\right)$$

$$= \frac{144}{s^5} - \frac{77}{s^2+121}$$

PART V : First Shift Theorem

17) $\mathcal{L}\{e^{2t}\cos 3t \cdot u(t)\}$

$a=2 \qquad b=3$

$$= \frac{s-2}{(s-2)^2+3^2}$$

$$= \frac{s-2}{(s-2)^2+9}$$

18) $\mathcal{L}\{e^{-7t}\sin(5t)\cdot u(t)\}$

$a=-7 \qquad b=5$

$$= \frac{5}{(s-a)^2+5^2}$$

$$= \frac{5}{(s-a)^2+25}$$

$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{at}u(t)$	$\frac{1}{s-a}$
$t^n e^{at}u(t)$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at)\cdot u(t)$	$\frac{a}{s^2+a^2}$
$\cos(at)\cdot u(t)$	$\frac{s}{s^2+a^2}$
$e^{at}\sin(bt)\cdot u(t)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)\cdot u(t)$	$\frac{a}{s^2-a^2}$
$\cosh(at)\cdot u(t)$	$\frac{s}{s^2-a^2}$
$e^{at}\sinh(bt)\cdot u(t)$	$\frac{b}{(s-a)^2-b^2}$
$e^{at}\cosh(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2-b^2}$

Alg. manip
in inv.
comp the square
eg. +2 -2

19) $\mathcal{L}\{(4e^{-2t}\sin(3t) - 2e^{8t}\cos(19t))\cdot u(t)\}$

$$= 4\,\mathcal{L}\{e^{-2t}\sin(3t)\} - 2\{e^{8t}\cos(19t)\}$$

$a=-2 \quad b=3 \qquad a=8 \quad b=19$

$$= 4\left(\frac{3}{(s--2)^2+3^2}\right) - 2\left(\frac{s-8}{(s-8)^2+19^2}\right)$$

$$= \frac{12}{(s+2)^2+9} - \frac{2s-16}{(s-8)^2+361}$$

20) $\mathcal{L}\{e^{-5t}t^6 + \frac{1}{8}e^t - 4e^{13t}\cosh(20t)\}$

$a=-5 \quad n=6 \qquad a=1 \qquad a=13 \quad b=20$

$$= \frac{6!}{(s+5)^{6+1}} + \frac{1}{8}\cdot\frac{1}{s-1} - 4\left(\frac{s-13}{(s-13)^2-20^2}\right)$$

$$= \frac{720}{(s+5)^7} + \frac{1}{8s-8} - \frac{4(s-13)}{(s-13)^2-400}$$

$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{at}u(t)$	$\frac{1}{s-a}$
$t^n e^{at}u(t)$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at)\cdot u(t)$	$\frac{a}{s^2+a^2}$
$\cos(at)\cdot u(t)$	$\frac{s}{s^2+a^2}$
$e^{at}\sin(bt)\cdot u(t)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)\cdot u(t)$	$\frac{a}{s^2-a^2}$
$\cosh(at)\cdot u(t)$	$\frac{s}{s^2-a^2}$
$e^{at}\sinh(bt)\cdot u(t)$	$\frac{b}{(s-a)^2-b^2}$
$e^{at}\cosh(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2-b^2}$

PART VI - INVERSE LAPLACE TRANSFORMS

$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{at}u(t)$	$\frac{1}{s-a}$
$t^n e^{at} u(t)$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at)\cdot u(t)$	$\frac{a}{s^2+a^2}$
$\cos(at)\cdot u(t)$	$\frac{s}{s^2+a^2}$
$e^{at}\sin(bt)\cdot u(t)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)\cdot u(t)$	$\frac{a}{s^2-a^2}$
$\cosh(at)\cdot u(t)$	$\frac{s}{s^2-a^2}$
$e^{at}\sinh(bt)\cdot u(t)$	$\frac{b}{(s-a)^2-b^2}$
$e^{at}\cosh(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2-b^2}$

21) $\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{2!}{s^{2+1}}\right\}$$

$$f(t) = \underline{\underline{t^2 u(t)}}$$

22) $f(t) = \mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+2^2}\right\}$$

$$f(t) = \underline{\underline{\cos(2t)\cdot u(t)}}$$

23) $F(s) = \frac{840}{s^6}$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{840}{s^6}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{k\cdot 5!}{s^{5+1}}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{k\cdot 120}{s^{5+1}}\right\}$$

⇒ We need it in this form, $\frac{5!}{s^{5+1}}$ or $\frac{120}{s^{5+1}}$

$$k \times 120 = 840$$

$$k = \frac{840}{120} = 7$$

$$= \mathcal{L}^{-1}\left\{\frac{7\cdot 120}{s^{5+1}}\right\}$$

$$= 7\,\mathcal{L}^{-1}\left\{\frac{5!}{s^{5+1}}\right\}$$

$$f(t) = \underline{\underline{7t^5\cdot u(t)}}$$

$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{at}u(t)$	$\frac{1}{s-a}$
$t^n e^{at}u(t)$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at)\cdot u(t)$	$\frac{a}{s^2+a^2}$
$\cos(at)\cdot u(t)$	$\frac{s}{s^2+a^2}$
$e^{at}\sin(bt)\cdot u(t)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)\cdot u(t)$	$\frac{a}{s^2-a^2}$
$\cosh(at)\cdot u(t)$	$\frac{s}{s^2-a^2}$
$e^{at}\sinh(bt)\cdot u(t)$	$\frac{b}{(s-a)^2-b^2}$
$e^{at}\cosh(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2-b^2}$

24) $F(s) = \frac{1}{s^2+9}$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2+3^2}\right\}$$

We need it in this form $\frac{3}{s^2+3^2}$

$$= \mathcal{L}^{-1}\left\{\frac{3}{3}\cdot\frac{1}{s^2+3^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{3}\cdot\frac{3}{s^2+3^2}\right\}$$

$$= \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\}$$

$$f(t) = \underline{\underline{\frac{1}{3}\sin(3t)\cdot u(t)}}$$

$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{at}u(t)$	$\frac{1}{s-a}$
$t^n e^{at}u(t)$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at)\cdot u(t)$	$\frac{a}{s^2+a^2}$
$\cos(at)\cdot u(t)$	$\frac{s}{s^2+a^2}$
$e^{at}\sin(bt)\cdot u(t)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)\cdot u(t)$	$\frac{a}{s^2-a^2}$
$\cosh(at)\cdot u(t)$	$\frac{s}{s^2-a^2}$
$e^{at}\sinh(bt)\cdot u(t)$	$\frac{b}{(s-a)^2-b^2}$
$e^{at}\cosh(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2-b^2}$

28) $F(s) = \dfrac{5}{3s-1}$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{5}{3s-1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{5}{3}\cdot\frac{1}{s-\frac{1}{3}}\right\}$$

$$= \frac{5}{3}\mathcal{L}^{-1}\left\{\frac{1}{s-\frac{1}{3}}\right\}$$

$$= \frac{5}{3}\cdot e^{\frac{1}{3}t}u(t)$$

$$f(t) = \underline{\underline{\frac{5}{3}e^{\frac{1}{3}t}\cdot u(t)}}$$

26) $F(s) = \dfrac{23s}{2s^2+18}$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{23s}{2s^2+18}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{23}{2}\cdot\frac{s}{s^2+9}\right\}$$

$$= \frac{23}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+3^2}\right\}$$

$$f(t) = \underline{\underline{\frac{23}{2}\cos(3t)\cdot u(t)}}$$

27) $\mathcal{L}^{-1}\left\{\dfrac{5s-20}{9(s-4)^2+1{,}089}\right\}$

$= \mathcal{L}^{-1}\left\{\dfrac{5}{9}\cdot\dfrac{s-4}{(s-4)^2+121}\right\}$

$= \dfrac{5}{9}\mathcal{L}^{-1}\left\{\dfrac{s-4}{(s-4)^2+11^2}\right\}$

$a=4 \qquad b=11$

$$f(t) = \frac{5}{9}e^{4t}\cosh(11t)\cdot u(t)$$

$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{at}u(t)$	$\frac{1}{s-a}$
$t^n e^{at}u(t)$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at)\cdot u(t)$	$\frac{a}{s^2+a^2}$
$\cos(at)\cdot u(t)$	$\frac{s}{s^2+a^2}$
$e^{at}\sin(bt)\cdot u(t)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2+b^2}$
* $\sinh(at)\cdot u(t)$	$\frac{a}{s^2-a^2}$
$\cosh(at)\cdot u(t)$	$\frac{s}{s^2-a^2}$
$e^{at}\sinh(bt)\cdot u(t)$	$\frac{b}{(s-a)^2-b^2}$
* $e^{at}\cosh(bt)\cdot u(t)$	$\frac{s-a}{(s-a)^2-b^2}$

28) $\mathcal{L}^{-1}\left\{\dfrac{3}{s^2-7}\right\}$

$= \mathcal{L}^{-1}\left\{\dfrac{3}{s^2-\sqrt{7}^2}\right\}$

Numerator needs to be $\sqrt{7}$

① Take out factor of 3

$= \mathcal{L}^{-1}\left\{3\cdot\dfrac{1}{s^2-\sqrt{7}^2}\right\}$

② $\times 1 \quad \left(\dfrac{\sqrt{7}}{\sqrt{7}}\right)$

$= \mathcal{L}^{-1}\left\{3\cdot\dfrac{\sqrt{7}}{\sqrt{7}}\cdot\dfrac{1}{s^2-\sqrt{7}^2}\right\}$

$= \mathcal{L}^{-1}\left\{\dfrac{3}{\sqrt{7}}\cdot\dfrac{\sqrt{7}}{s^2-\sqrt{7}^2}\right\}$

$$= \frac{3}{\sqrt{7}} \mathcal{L}^{-1}\left\{ \frac{\sqrt{7}}{s^2 - \sqrt{7}^2} \right\}$$

$$a = \sqrt{7}$$

$$f(t) = \frac{3}{\sqrt{7}} \sinh(\sqrt{7}t) \cdot u(t)$$

$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{at} u(t)$	$\frac{1}{s-a}$
$t^n e^{at} u(t)$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at) \cdot u(t)$	$\frac{a}{s^2+a^2}$
$\cos(at) \cdot u(t)$	$\frac{s}{s^2+a^2}$
$e^{at} \sin(bt) \cdot u(t)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at} \cos(bt) \cdot u(t)$	$\frac{s-a}{(s-a)^2+b^2}$
* $\sinh(at) \cdot u(t)$	$\frac{a}{s^2-a^2}$
$\cosh(at) \cdot u(t)$	$\frac{s}{s^2-a^2}$
$e^{at} \sinh(bt) \cdot u(t)$	$\frac{b}{(s-a)^2-b^2}$
$e^{at} \cosh(bt) \cdot u(t)$	$\frac{s-a}{(s-a)^2-b^2}$